SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

TO FACTOR A PERFECT SQUARE TRINOMIAL:

1. Find the positive square roots of each of the two perfect square terminologies
2. Connect the two square roots with the sign of the numerical coefficient of the remaining term (the second term) of the given trinomial
3. Indicate that this binomial is used twice as a factor

QUADRATIC EQUATION

$$x^2 + bx + (\frac{b}{2})^2 = [x + \frac{b}{2}]^2$$

COMPLETING THE SQUARE:

In an expression of the form x^2 + bx or x^2 - bx, add the constant term $(\frac{b}{2})^2$ to complete the square

COMPLETING THE SQUARE:

EXAMPLE:

$x^2 + 14x$ ________

SOLUTION

a. To complete the square, add $(\frac{14}{2})^2$ or 49
b. Complete the square: $x^2 + 14x + 49$
c. Factored Form: $(x+7)^2$

COMPLETING THE SQUARE:

Example: $x^2 + 24x = 36$

$x^2 + 24x + 144 = 36 + 144$ (Addition Property of Equality)

$(x+12)^2 = 180$ (Factor the left-hand side and add the numbers on the right-hand side)

$x + 12 = \pm\sqrt{180}$ (Use the Square Root Principle)

$X = -12 \pm\sqrt{180}$ (Subtract 12 from both sides)

COMPLETING THE SQUARE:

$x = -12 \pm\sqrt{36.5}$ (Factor the Radicand)

$x = -12 \pm 6\sqrt{5}$ (Simplify the radical)

There are two solutions in this example:

$x = -12 + 6\sqrt{5}$

$x = -12 - 6\sqrt{5}$

STEPS IN SOLVING QUADRATIC EQUATIONS COMPLETING THE SQUARE:

1. Transpose all terms containing the unknown to the left side of the equation and the constant term to the right side, if necessary
2. Divide each term of the equation by the numerical coefficient of the x^2 term, if necessary. This will change the equation to the form of $x^2 + bx = c$

STEPS IN SOLVING QUADRATIC EQUATIONS COMPLETING THE SQUARE:

3. Divide the coefficient of x by 2, square it, then add the result to both sides of the equation
4. Factor the left side of the equation. This is a perfect square trinomial. Simplify the right side
5. Take the square root of the expression on both sides of the equation. Write the ± sign before the square root at the right side

STEPS IN SOLVING QUADRATIC EQUATIONS COMPLETING THE SQUARE:

6. Equate the square root on the left side expression to the positive square root on the right side in step 5. Solve for the first root. Then, find the second root of the equation

7. Equate the square root of the left side expression to the negative square root of the right side in step 5. Solve for the second root.

STEPS IN SOLVING QUADRATIC EQUATIONS COMPLETING THE SQUARE:

8. Check each root by substituting it to the the original equaton

EXAMPLE:

$2x^2 + 4x + 1 = 0$

Step 1: Transpose the constant term to the right side

($2x^2 + 4x = -1$)

Step 2: Make the coefficient of x^2 equal to 1 by multiplying all the terms by $\frac{1}{2}$

($x^2 + 2x = -\frac{1}{2}$)

EXAMPLE:

Step 3: Complete the square by adding the square of half the coefficient of x to both sides of the equation. This is

$(\frac{2}{2})^2$ or 1.

$(x^2 + 2x + 1 = -\frac{1}{2} + 1)$

Step 4: Factor the left side and perform the operation on the right side

$(x + 1)^2 = \frac{1}{2}$

EXAMPLE:

Step 5: Apply the Square Root Principle

$(x + 1 = \pm\sqrt{\frac{1}{2}})$

Step 6: Simplify the radical

$(x + 1 = \pm\frac{\sqrt{2}}{2})$

Step 7: Solve for x by subtracting one from both sides

$(x = 1 \pm\frac{\sqrt{2}}{2})$

EXAMPLE:

$x = -1 + \frac{\sqrt{2}}{2}$

$= \frac{-2+\sqrt{2}}{2}$

$x = -1 - \frac{\sqrt{2}}{2}$

$= \frac{-2-\sqrt{2}}{2}$